SUPPLEMENTAL MATERIAL

New Robust Approach for the Globally Optimal Design of Fired Heaters

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S1: Protocol to find the bounds for U_{conv} and $LMTD$

[1] Compute the bounds of the enthalpy of the radiation section entrance:

$$h_{fb}^{UB} = \hat{h}_{flame}^{gas} + \widehat{Cp}_{gas}\left(T_{fb}^{UB} - \hat{T}_{flame}\right) \quad \text{(S1-1)}$$

$$h_{fb}^{LB} = \hat{h}_{flame}^{gas} + \widehat{Cp}_{gas}\left(T_{fb}^{LB} - \hat{T}_{flame}\right) \quad \text{(S1-2)}$$

Here, $\hat{h}_{flame}^{gas}$ and $\hat{T}_{flame}$ are the enthalpy and temperature of the flame in the burner, respectively. They are parameters because the excess air is fixed. In turn, T_{fb}^{UB} and T_{fb}^{UB} are calculated during the proxy set trimming of the radiant section.

[2] Compute the bounds of the enthalpy of the stack entrance:

$$h_{s}^{UB} = h_{fb}^{UB} - \left(1 - \widehat{pLoss}_{Conv}\right)Q_{Conv}^{LB}/M_{gas}^{UB} \quad \text{(S1-3)}$$

$$h_{s}^{LB} = h_{fb}^{LB} - \left(1 - \widehat{pLoss}_{Conv}\right)Q_{Conv}^{UB}/M_{gas}^{LB} \quad \text{(S1-4)}$$

[3] Calculate the inlet stack temperature bounds:

$$T_{s}^{UB} = \left(\frac{h_{s}^{UB} - h_{g}}{cpg_{1}}\right) + T_{fb}^{UB} \quad \text{(S1-5)}$$

$$T_{s}^{LB} = \left(\frac{h_{s}^{LB} - h_{g}}{cpg_{1}}\right) + T_{fb}^{LB} \quad \text{(S1-6)}$$

[4] Obtain the bounds on *LMTD*:

$$LMTD^{UB} = \frac{\left(T_{fb}^{UB} - T_{c}^{LB}\right) - \left(T_{s}^{LB} - \hat{T}_{i}\right)}{ln\left(\frac{T_{fb}^{LB} - T_{c}^{UB}}{T_{s}^{UB} - \hat{T}_{i}}\right)} \quad \text{(S1-7)}$$

$$LMTD^{LB} = \frac{\left(T_{fb}^{LB} - T_{c}^{UB}\right) - \left(T_{s}^{UB} - \hat{T}_{i}\right)}{ln\left(\frac{T_{fb}^{UB} - T_{c}^{LB}}{T_{s}^{LB} - \hat{T}_{i}}\right)} \quad \text{(S1-8)}$$

[5] Obtain the heat transfer coefficient inside the tubes using the Sieder-Tate correlation

$$h_{Ci}^{HT} = 0.023 \frac{\hat{k}_{oil}}{\left[d_{i}^{conv}\right]^{1.8}} \left\{\frac{4\hat{M}_{Oil}}{N_{passes}\,\pi\,\hat{\mu}_{oil}}\right\}^{0.8} \widehat{Pr}_{oil}^{0.33} \quad \text{(S1-9)}$$

[6] Obtain the bounds on the *j* factor for a triangular layout:

$$C_{1.LB} = 0.091\left(\frac{d_o^{conv}\, G_{LB}}{\hat{\mu}_{gas}}\right)^{-0.25} \tag{S1-10}$$

$$C_{1.UB} = 0.091\left(\frac{d_o^{conv}\, G_{UB}}{\hat{\mu}_{gas}}\right)^{-0.25} \tag{S1-11}$$

$$j_{LB} = C_{1,LB}C_3C_5\left(\frac{d_f}{d_o^{conv}}\right)^{0.5} \tag{S1-12}$$

$$j_{UB} = C_{1,UB}C_3C_5\left(\frac{d_f}{d_o^{conv}}\right)^{0.5} \tag{S1-13}$$

[7] Obtain the convective heat transfer coefficient for the flue gas flow around the finned surface:

$$h_{Co}^{HT,LB} = j_{LB}\ \widehat{Cp}_{gas}\, G_{LB}\, Pr_{gas}^{-0.67} \tag{S1-14}$$

$$h_{Co}^{HT,UB} = j_{UB}\ \widehat{Cp}_{gas}\, G_{UB}\, Pr_{gas}^{-0.67} \tag{S1-15}$$

[8] Calculate the bounds on fin efficiency:

$$\frac{1}{h_{LB}^{'}} = \frac{1}{h_{Co}^{HT,LB}} + \widehat{Rf}_{gas} \tag{S1-16}$$

$$\frac{1}{h_{UB}^{'}} = \frac{1}{h_{Co}^{HT,UB}} + \widehat{Rf}_{gas} \tag{S1-17}$$

$$l_{fe} = l_f\left(1 + \frac{t_f}{2l_f}\right)\left[1 + 0.35\ln\left(\frac{d_f}{d_o^{conv}}\right)\right] \tag{S1-18}$$

$$mf_{LB} = \sqrt{\left(\frac{2\, h_{LB}^{'}}{\hat{k}_{fin} t_f}\right)} \tag{S1-19}$$

$$mf_{UB} = \sqrt{\left(\frac{2\, h_{UB}^{'}}{\hat{k}_{fin}\, t_f}\right)} \tag{S1-20}$$

$$\eta f_{LB} = \frac{\tanh(mf_{LB}\, l_{fe})}{mf_{UB}\, l_{fe}} \tag{S1-21}$$

$$\eta f_{UB} = \frac{\tanh(mf_{UB}\, Lfe)}{mf_{LB}\, Lfe} \tag{S1-22}$$

[9] Obtain the overall efficiency of the finned surface:

$$\eta t_{LB} = \left(\frac{Aot - Aof}{Aot}\right) + \eta f_{LB}\left(\frac{Aof}{Aot}\right) \tag{S1-23}$$

$$\eta t_{UB} = \left(\frac{Aot - Aof}{Aot}\right) + \eta f_{UB}\left(\frac{Aof}{Aot}\right) \tag{S1-24}$$

[10] Obtain the upper and lower limit of U_{Conv}:

$$U_{Conv,LB} = \frac{1}{\left(\frac{1}{h_{Ci}^{HT}}+\widehat{Rf}t_{max}\right)\left(\frac{Aot}{\pi\, d_i}\right)+\frac{Aot\ln\left(\frac{d_o}{d_i}\right)}{2\,\pi\,\hat{k}_S}+\frac{1}{\eta t_{LB}\, h_{Co,LB}^{HT}}+\frac{\widehat{Rf}_{gas}}{\eta t_{LB}}} \tag{S1-25}$$

$$U_{Conv,UB} = \frac{1}{\left(\frac{1}{h_{Ci}^{HT}}+\widehat{Rf}t_{max}\right)\left(\frac{Aot}{\pi\, d_i}\right)+\frac{Aot\ln\left(\frac{d_o}{d_i}\right)}{2\,\pi\,\hat{k}_S}+\frac{1}{\eta t_{UB}\, h_{Co,UB}^{HT}}+\frac{\widehat{Rf}_{gas}}{\eta t_{UB}}} \tag{S1-26}$$

S2: Lower Bound on friction losses in the stack equation

[1] Compute the upper gas density (we use ideal gas law with molecular weight corresponding to burning natural gas with fixed excess air):

$$\rho_{fb}^{UB} = \frac{P}{R\, T_{fb}^{LB}} MW_{avg} \tag{S2-1}$$

[2] Compute the density of the gas at the convection section outlet:

$$\rho_S^{LB} = \frac{P}{R\, T_S^{UB}} MW_{avg} \tag{S2-2}$$

[3]Compute the mean density of the gas in the convection section:

$$\rho_m^{LB} = \frac{P}{R\left(\frac{T_S^{UB}+T_{fb}^{UB}}{2}\right)} MW_{avg} \tag{S2-3}$$

$$\rho_m^{UB} = \frac{P}{R\left(\frac{T_S^{LB}+T_{fb}^{LB}}{2}\right)} MW_{avg} \tag{S2-4}$$

[4]Calculate the friction factor bounds:

$$C_{2,LB} = 0.075 + 1.85\left(\frac{d_o^{conv}\, G_{UB}}{\hat{\mu}_{gas}}\right)^{-0.3} \tag{S2-5}$$

$$f_{LB} = C_{2,LB}C_4C_6\left(\frac{d_f}{d_o^{conv}}\right) \tag{S2-6}$$

[5]Obtain the pressure drop across the finned horizontal tubes in the convection section:

$$a_{LB} = \frac{1+\beta^2}{4\ Nr_{conv}}\rho_m^{LB}\left(\frac{1}{\rho_{fb}^{UB}} - \frac{1}{\rho_S^{LB}}\right) \tag{S2-7}$$

$$\Delta p_f^{LB} = (f_{LB} + a_{LB}) Nr_{conv} \frac{G_{LB}^2}{\rho_m^{UB}} \qquad \text{(S2-8)}$$

[6] Obtain the pressure drop across the shield tubes:

$$\Delta p_{shield}^{LB} = 0.2 \frac{G_{LB}^2}{2\rho_{fb}^{UB}} \qquad \text{(S2-9)}$$

[7] Obtain the pressure drop in the convection section:

$$\Delta p_{conv}^{LB} = \Delta p_{shield}^{LB} + \Delta p_f^{LB} \qquad \text{(S2-10)}$$

[8] Compute the lower bound on the stack mass flux:

$$G_{stack}^{LB} = \left(\frac{M_{gas}^{LB}}{\frac{\pi D_S^2}{4}} \right) \qquad \text{(S2-11)}$$

[9] Compute the pressure drop along the stack:

$$\Delta p_{stack}^{LB} = 2.76 \cdot 10^{-5} \left(\frac{\left(G_{stack}^{LB}\right)^2 \left(\frac{T_S^{LB} + T_a}{2} \right)}{D_s} \right) H_s \qquad \text{(S2-12)}$$

[10] Compute the minor losses associated with the flow along the stack:

$$\Delta p_{minor}^{LB} = 3 \frac{\left(G_{stack}^{LB}\right)^2}{2\rho_{stack}} \qquad \text{(S2-13)}$$

[11] Obtain the friction losses:

$$\left[\sum F_{LB}\right]^{LB} = \Delta p_{conv}^{LB} + \Delta p_{stack}^{LB} + \Delta p_{minor}^{LB} \qquad \text{(S2-14)}$$

S3: Full Fired Heater Model

All geometry variables are fixed. They correspond to the candidates that survived the Set Trimming procedure. The values of the variables needed to obtain the total cost for a fixed geometry are obtained by solving the following system of equations:

$$Q_{rad}\left(1 - \widehat{percLoss}_{Rad}\right) = \alpha\, A_{cp}\, F\, \hat{\sigma}\left(T_{fb}^4 - T_w^4\right) + \widehat{hcr} A_{rad}\left(T_{fb} - T_w\right) \qquad \text{(S3-1)}$$

$$\left\{\begin{aligned} F = \hat{c}_{F,1}\, log(e_r) + \left(\frac{\hat{c}_{F,2}}{e} + \hat{c}_{F,3}\right)\left\{e_r\left(\left[\frac{2\,W_{rad}\,H_{rad} + 2\,E_l\,(W_{rad}+H_{rad})}{\hat{\alpha}A_{cp}}\right] - 1\right)\right\} \\ +\hat{c}_{F,4}\, exp(e_r) + \hat{c}_{F,5}\left\{e_r\,\left(\left[\frac{2\,W_{rad}\,H_{rad} + 2\,E_l\,(W_{rad}+H_{rad})}{\hat{\alpha}A_{cp}}\right] - 1\right)\right\}^{1.5} + \hat{c}_{F,6} \end{aligned}\right\} \quad \text{(S3-1)}$$

$$e_r = \hat{c}_{e,1}T_{fb} + \hat{c}_{e,2}PL + \hat{c}_{e,3}PL^2 + \hat{c}_{e,4} \quad \text{(S3-2)}$$

$$Q_{Rad}\left(1 - \widehat{percLoss}_{Rad}\right) = \widehat{M}_{Oil}\left(\hat{h}_o^{oil} - \widehat{co}_{h,1}(T_c)^2 - \widehat{co}_{h,2}T_c - \widehat{co}_{h,3}\right) \quad \text{(S3-3)}$$

$$Q_{rad}\left(1 - \widehat{percLoss}_{Rad}\right) = \left[\frac{1}{\widehat{Rf}_{gas} + \frac{d_o^{rad}\ln(d_o^{rad}/d_i^{rad})}{2\hat{k}_s} + \widehat{Rft}_{max}\left(\frac{d_o^{rad}}{d_i^{rad}}\right) + \frac{1}{hct}\left(\frac{d_o^{rad}}{d_i^{rad}}\right)}\right] A_{rad}\left(T_w - \left(\frac{(\hat{T}_O + T_C)}{2}\right)\right) \quad \text{(S3-4)}$$

$$M_{gas} = \frac{Q_{Rad}\,(1 - \widehat{percLoss}_{Rad})}{\widehat{cpg}_1(\hat{T}_{flame} - T_{fb})} \quad \text{(S3-5)}$$

$$T_s = T_{fb} - \left(\frac{Q_{conv}(1 - \widehat{percLoss}_{Conv})}{M_{gas}\ \widehat{Cp}_{gas}}\right) \quad \text{(S3-6)}$$

$$Q_{conv}\left(1 - perc\hat{L}ossconv\right) = \hat{Q}_{oil} - Q_{rad}\left(1 - perc\hat{L}ossconv\right) \quad \text{(S3-7)}$$

$$Q_{conv}\left(1 - \widehat{percLoss}_{Conv}\right) = U_{conv}A_{conv}LMTD \quad \text{(S3-8)}$$

$$LMTD = \left[(T_{fb} - T_c) - (T_s - \hat{T}_i)\right]/\,ln\left\{(T_{fb} - T_c)/(T_s - \hat{T}_i)\right\} \quad \text{(S3-9)}$$

$$U_{Conv} = \frac{1}{\left(\frac{1}{h_{Ci}^{HT}} + \widehat{Rft}_{max}\right)\left(\frac{Aot}{\pi\, d_i}\right) + \frac{Aot\ln\left(\frac{d_O}{d_i}\right)}{2\,\pi\,\hat{k}_t} + \frac{1}{\eta t\, h_{Co}^{HT}} + \frac{\widehat{Rfgas}}{\eta t}} \quad \text{(S3-10)}$$

$$h_{Ci}^{HT} = 0.023\,\frac{\hat{k}_{oil}}{d_i^{1.8}}\left\{\frac{4\widehat{M}_{Oil}}{N_{passes}\,\pi\,\hat{\mu}_{oil}}\right\}^{0.8}\widehat{Pr}^{0.33} \quad \text{(S3-11)}$$

$$h_{Co}^{HT} = j\ \widehat{Cp}_{gas}\ G\ \ \widehat{Pr}_{gas}^{-0.67} \quad \text{(S3-12)}$$

$$G = \frac{M_{gas}}{A_s} \quad \text{(S3-13)}$$

$$\eta t = \left(\frac{Aot - Aof}{Aot}\right) + \eta f\left(\frac{Aof}{Aot}\right) \quad \text{(S3-14)}$$

$$\eta f = \frac{\tanh(mf\ l_{fe})}{mf\ l_{fe}} \quad \text{(S3-15)}$$

$$mf = \sqrt{\left(\frac{2\,h'}{\hat{k}_{fin}\,t_f}\right)} \quad \text{(S3-16)}$$

$$l_{fe} = l_f\ \left(1 + \frac{t_f}{2L_f}\right)\left[1 + 0.35\ln\left(\frac{d_f}{d_o}\right)\right] \quad \text{(S3-17)}$$

$$\frac{1}{h'} = \frac{1}{h_{Co}^{HT}} + \widehat{Rfgas} \tag{S3-18}$$

$$j = C_1 C_3 C_5 \left(\frac{d_f}{d_o}\right)^{0.5} \tag{S3-19}$$

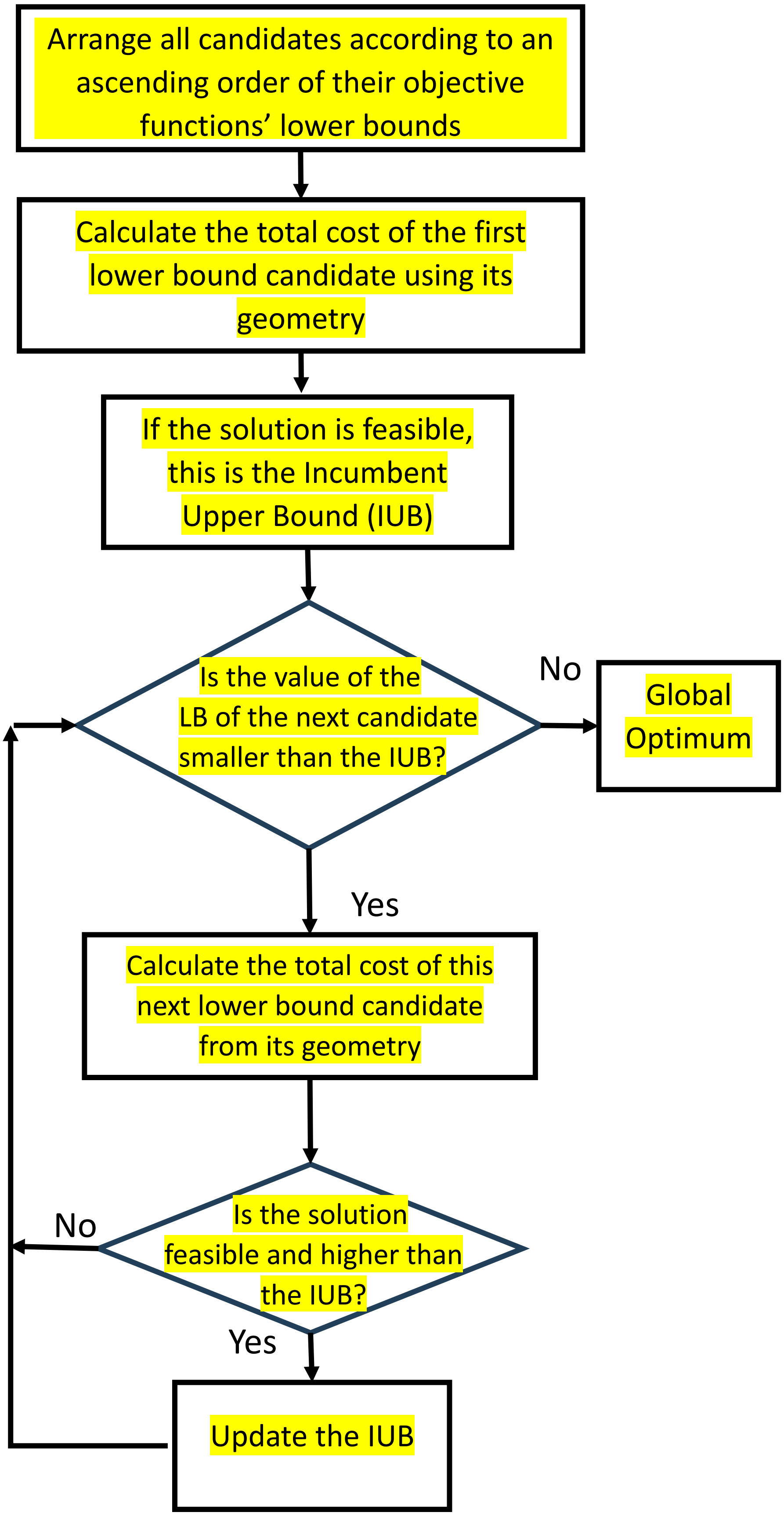
$$C_1 = 0.091 Re^{-0.25} \quad = \quad 0.091 \left(\frac{d_o\ G}{\hat{\mu}_{gas}}\right)^{-0.25} \tag{S3-20}$$

$$C_3 = 0.35 + 0.65 \exp\left[-\frac{0.125(d_f - d_o)}{s}\right] \tag{S3-21}$$

$$C_5 = 0.7 + \left[0.7 - 0.8\, e^{-0.15(Nr_{conv})^2}\right] e^{-\left(\frac{d_c^{conv,v}}{d_c^{conv,h}}\right)} \tag{S3-22}$$

$$s = \frac{1}{N_f} - t_f \tag{S3-23}$$

S4: Smart Enumeration Flowchart



S5: Proof of the Uniqueness of the Solution of the Full Model

Physical considerations:

The system of nonlinear equations for the evaluation of a design candidate with a given geometry corresponds to the identification of the fuel flow rate that provides the desired heat load for heating the oil from the inlet temperature to the outlet temperature.

An increase of the fuel flow rate always yields an increase of the heat load, because it implies in higher flue gas temperature and flow rate, i.e. higher heat transfer driving force and heat transfer coefficient.

Then, considering that the variation of the heat load with fuel flow rate is always positive, as explained above (a monotonic behavior), there is only one value of fuel flow rate that attains the desired heat load (lower values of fuel flow rate would imply in lower heat loads than the desired value and higher values of fuel flow rates would imply higher heat loads than the desired value). This behavior can be illustrated by the existent control loops in fired heaters, where the outlet temperature of the oil stream is controlled by a valve that manipulates the fuel flow rate.

Mathematical proof by contradiction:

Consider two different solutions (A and B). Consider the temperature of the oil entering the radiation section (T_c).

Assume $T_{c,A} > T_{c,B}$. Because the r.h.s of equation (30) in the main article is monotone decreasing, we have $Q_{rad,A} < Q_{rad,B}$.

In addition, from of equation (32) in the main article, we get

$$Q_{rad}\left(1-\widehat{pLoss}_{rad}\right) = \alpha\, A_{cp}\, F\, \hat{\sigma}\left(T_{fb}^4 - T_w^4\right) + \widehat{hcr} A_{rad}\left(T_{fb} - T_w\right) \quad \text{(S5-24)}$$

$$F = \hat{c}_{F,1}\ln(e_r) + \left(\hat{c}_{F,2} + \hat{c}_{F,3} e_r\right)\Omega + \hat{c}_{F,4}\exp(e_r) + \hat{c}_{F,5}\{e_r\,\Omega\}^{1.5} + \hat{c}_{F,6} \quad \text{(S5-25)}$$

$$e_r = \hat{c}_{e,1} T_{fb} + \Psi \quad \text{(S5-26)}$$

where

$$\Omega = \frac{2\,W_{rad}\,H_{rad} + 2\,E_l\,(W_{rad} + H_{rad})}{\hat{\alpha} A_{cp}} - 1 \quad \text{(S5-27)}$$

$$\Psi = \hat{c}_{e,2} PL + \hat{c}_{e,3} PL^2 + \hat{c}_{e,4} \quad \text{(S5-28)}$$

Replacing, we get

$$Q_{rad}\left(1-\widehat{pLoss}_{rad}\right) = \alpha\, A_{cp}\left\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb} + \Psi\right) + \left(\hat{c}_{F,2} + \hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb} + \Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb} + \Psi\right) + \hat{c}_{F,5}\left\{\left[\hat{c}_{e,1}T_{fb} + \Psi\right]\Omega\right\}^{1.5} + \hat{c}_{F,6}\right\}\hat{\sigma}\left(T_{fb}^4 - T_w^4\right) + \widehat{hcr} A_{rad}\left(T_{fb} - T_w\right) \quad \text{(S5-29)}$$

From of equation (30) and (32) in the main article

$$T_w = \frac{\widehat{M}_{Oil}}{U_{rad,w}\,A_{rad}}\left(\hat{h}_o^{oil} - \widehat{co}_{h,1}(T_c)^2 - \widehat{co}_{h,2}T_c - \widehat{co}_{h,3}\right) + \left(\frac{(\hat{T}_o+T_c)}{2}\right) = \Theta_1 - \Theta_2(T_c)^2 - \Theta_3 T_c \qquad \text{(S5-30)}$$

where

$$\Theta_1 = \frac{\widehat{M}_{Oil}}{U_{rad,w}\,A_{rad}}\left(\hat{h}_o^{oil} - \widehat{co}_{h,3}\right) + \frac{\hat{T}_o}{2} \qquad \text{(S5-31)}$$

$$\Theta_2 = \frac{\widehat{M}_{Oil}}{U_{rad,w}\,A_{rad}}\widehat{co}_{h,1} \qquad \text{(S5-32)}$$

$$\Theta_3 = \frac{\widehat{M}_{Oil}}{U_{rad,w}\,A_{rad}}\widehat{co}_{h,2} - \frac{1}{2} \qquad \text{(S5-33)}$$

Substituting

$$Q_{rad}\left(1-\widehat{pLoss}_{rad}\right) = \alpha\, A_{cp}\left\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2}+\hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\left\{\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\Omega\right\}^{1.5} + \hat{c}_{F,6}\right\}\hat{\sigma}\left(T_{fb}^4 - \left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]^4\right) + \widehat{hcr}A_{rad}\left(T_{fb}-\left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]\right) \qquad \text{(S5-34)}$$

$$\delta Q_{rad}\frac{(1-\widehat{pLoss}_{rad})}{\alpha\, A_{cp}\hat{\sigma}} = \left\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2}+\hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\left\{\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\Omega\right\}^{1.5} + \hat{c}_{F,6}\right\}\left(T_{fb}^4 - \left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]^4\right) + \frac{\widehat{hcr}A_{rad}}{\alpha\, A_{cp\hat{\sigma}}}\left(\delta T_{fb} + 2\Theta_2T_c\,\delta T_c + \Theta_3\delta T_c\right) \qquad \text{(S5-35)}$$

$$\delta Q_{rad}\frac{(1-\widehat{pLoss}_{rad})}{\alpha\, A_{cp}\hat{\sigma}} = \delta\left\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2}+\hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\left\{\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\Omega\right\}^{1.5} + \hat{c}_{F,6}\right\}\left(T_{fb}^4 - \left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]^4\right) + \left\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2}+\hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\left\{\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\Omega\right\}^{1.5} + \hat{c}_{F,6}\right\}\left(4T_{fb}^3\delta T_{fb} + 4\left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]^3\right)\left(2\Theta_2T_c\,\delta T_c + \Theta_3\delta T_c\right) + \frac{\widehat{hcr}A_{rad}}{\alpha\, A_{cp\hat{\sigma}}}\left(\delta T_{fb} + 2\Theta_2T_c\,\delta T_c + \Theta_3\delta T_c\right) \qquad \text{(S5-36)}$$

$$\delta Q_{rad}\frac{(1-\widehat{pLoss}_{rad})}{\alpha\, A_{cp}\hat{\sigma}} = \left\{\frac{\hat{c}_{F,1}}{\left((\hat{c}_{e,1}T_{fb}+\Psi\right)}\hat{c}_{e,1}\delta T_{fb} + \hat{c}_{F,3}\hat{c}_{e,1}\Omega\,\delta T_{fb} + \hat{c}_{F,4}\,\hat{c}_{e,1}\exp\left(T_{fb}+\Psi\right)\delta T_{fb} + 1.5\,\hat{c}_{F,5\hat{c}_{e,1}}\left\{\left[T_{fb}+\Psi\right]\Omega\right\}^{0.5}\delta T_{fb}\right\}\left(T_{fb}^4 - \left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]^4\right) + \left\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2}+\hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\left\{\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\Omega\right\}^{1.5} + \hat{c}_{F,6}\right\}\left(4T_{fb}^3\delta T_{fb} + 4\left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]^3\right)\left(2\Theta_2T_c\,\delta T_c + \Theta_3\delta T_c\right) + \frac{\widehat{hcr}A_{rad}}{\alpha\, A_{cp\hat{\sigma}}}\left(\delta T_{fb} + 2\Theta_2T_c\,\delta T_c + \Theta_3\delta T_c\right) \qquad \text{(S5-37)}$$

$$\delta Q_{rad}\frac{(1-\widehat{pLoss}_{rad})}{\alpha\, A_{cp}\hat{\sigma}} = \left\{\frac{\hat{c}_{F,1}}{\left((\hat{c}_{e,1}T_{fb}+\Psi\right)}\hat{c}_{e,1} + \hat{c}_{F,3}\hat{c}_{e,1}\Omega + \hat{c}_{F,4}\,\hat{c}_{e,1}\exp\left(T_{fb}+\Psi\right) + 1.5\,\hat{c}_{F,5\hat{c}_{e,1}}\left\{\left[T_{fb}+\Psi\right]\Omega\right\}^{0.5}\right\}\left(T_{fb}^4 - \left[\Theta_1-\Theta_2(T_c)^2-\Theta_3T_c\right]^4\right)\delta T_{fb} + \left\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2}+\hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\left\{\left[\hat{c}_{e,1}T_{fb}+\right.\right.$$

$$\Psi]\,\Omega\}^{1.5} + \hat{c}_{F,6}\Big\}\left(4T_{fb}^3\delta T_{fb} + 4\,[\Theta_1 - \Theta_2(T_c)^2 - \Theta_3 T_c]^3\right)(2\Theta_2 T_c\,\delta T_c + \Theta_3\delta T_c) + \frac{\widehat{hcr}A_{rad}}{\alpha\,A_{cp\hat{\sigma}}}\left(\delta T_{fb} + 2\Theta_2 T_c\,\delta T_c + \Theta_3\delta T_c\right) \quad \text{(S5-38)}$$

$$\delta Q_{rad}\frac{(1-\widehat{pLoss}_{rad})}{\alpha\,A_{cp}\hat{\sigma}} = \Lambda_1\delta T_{fb} + \Lambda_2\delta T_c \quad \text{(S5-39)}$$

where

$$\Lambda_1 = \left\{\frac{\hat{c}_{F,1}}{(\hat{c}_{e,1}T_{fb}+\Psi)}\hat{c}_{e,1} + \hat{c}_{F,3}\hat{c}_{e,1}\Omega + \hat{c}_{F,4}\,\hat{c}_{e,1}\exp\left(T_{fb}+\Psi\right) + 1.5\,\hat{c}_{F,5\hat{c}_{e,1}}\{[T_{fb} + \Psi]\,\Omega\}^{0.5}\right\}\left(T_{fb}^4 - [\Theta_1 - \Theta_2(T_c)^2 - \Theta_3 T_c]^4\right) + \Big\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2} + \hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\{[\hat{c}_{e,1}T_{fb}+\Psi]\,\Omega\}^{1.5} + \hat{c}_{F,6}\Big\}4T_{fb}^3 + \frac{\widehat{hcr}A_{rad}}{\alpha\,A_{cp\hat{\sigma}}} \quad \text{(S5-40)}$$

$$\Lambda_2 = \Big\{\hat{c}_{F,1}\ln\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \left(\hat{c}_{F,2} + \hat{c}_{F,3}\left[\hat{c}_{e,1}T_{fb}+\Psi\right]\right)\Omega + \hat{c}_{F,4}\exp\left(\hat{c}_{e,1}T_{fb}+\Psi\right) + \hat{c}_{F,5}\{[\hat{c}_{e,1}T_{fb}+\Psi]\,\Omega\}^{1.5} + \hat{c}_{F,6}\Big\}\left(4\,[\Theta_1 - \Theta_2(T_c)^2 - \Theta_3 T_c]^3\right)(2\Theta_2 T_c + \Theta_3) + \frac{\widehat{hcr}A_{rad}}{\alpha\,A_{cp\hat{\sigma}}}(2\Theta_2 T_c + \Theta_3) \quad \text{(S5-41)}$$

Or

$$\delta T_{fb} = \delta Q_{rad}\frac{(1-\widehat{pLoss}_{rad})}{\alpha\,A_{cp}\hat{\sigma}\Lambda_1} - \frac{\Lambda_2}{\Lambda_1}\delta T_c \quad \text{(S5-42)}$$

We conclude that $T_{fb,A} < T_{fb,B}$, which is also intuitive. We now continue with equation (35) from the main article, which renders

$$Q_{conv,A} - Q_{conv,B} = \left(Q_{rad,B} - Q_{rad,A}\right)\frac{(1-\widehat{pLoss}_{rad})}{(1-\widehat{pLoss}_{conv})} \quad \text{(S5-43)}$$

Then $Q_{conv,A} > Q_{conv,B}$.

We turn our attention to G_{conv}. Using (39) and (44) from the main article, using the inequalities obtained for Q_{rad} and T_{fb}, we get

$$G_{conv,A} = \frac{Q_{rad,A}(1-\widehat{pLoss}_{rad})}{A_s\,\widehat{Cp}_{gas}(\hat{T}_{flame}-T_{fb,A})} < \frac{Q_{rad,B}(1-\widehat{pLoss}_{rad})}{A_s\,\widehat{Cp}_{gas}(\hat{T}_{flame}-T_{fb,A})} < \frac{Q_{rad,B}(1-\widehat{pLoss}_{rad})}{A_s\,\widehat{Cp}_{gas}(\hat{T}_{flame}-T_{fb,B})} = G_{conv,B} \quad \text{(S5-44)}$$

Then, $G_{conv,A} < G_{conv,B}$.

We now look into equation (41) in the main article, we have

$$U_{Conv} = \frac{1}{\Theta + \frac{1}{\eta t\, h_{Co}^{HT}}} \quad \text{(S5-45)}$$

where

$$\Theta = \left(\frac{1}{h_{Ci}^{HT}} + \widehat{Rf}t_{max}\right)\left(\frac{Aot}{\pi\, d_i^{rad}}\right) + \frac{Aot\ln\left(\frac{d_o^{conv}}{d_i^{conv}}\right)}{2\,\pi\,\hat{k}_t} + \frac{\widehat{Rf}_{gas}}{\eta t} \quad \text{(S5-46)}$$

In turn equations (43), (54) and (55) from the main article, give

$$h_{Co}^{HT} = \Upsilon \;\; G_{conv}^{0.75} \qquad \text{(S5-47)}$$

where

$$\Upsilon = 0.091 \left(\frac{d_o^{conv}}{\hat{\mu}_{gas}}\right)^{-0.25} C_3 C_5 \left(\frac{d_f}{d_o^{conv}}\right)^{0.5} \;\widehat{Cp}_{gas} \;\; \widehat{Pr}_{gas}^{-0.67} \qquad \text{(S5-48)}$$

Thus, $h_{Co,A}^{HT} < h_{Co,B}^{HT}$, and using equation (22) from the main article, we get $U_{Conv,A} < U_{Conv,B}$

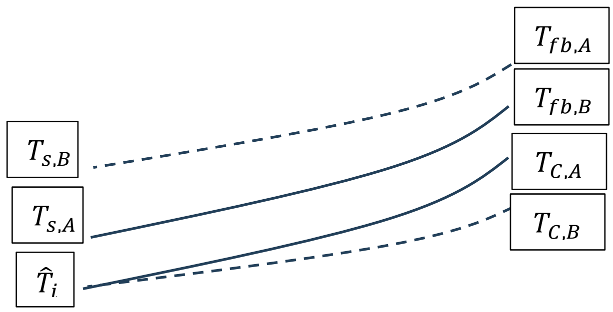
Now, we concentrate on equation (40) from the main article, to write

$$T_s = T_{fb} - \frac{Q_{conv}(1-\widehat{pLoss}_{conv})}{G_{conv} A_s} \qquad \text{(S5-49)}$$

Then, because, $T_{fb,A} < T_{fb,B}$, $Q_{conv,A} > Q_{conv,B}$ and $G_{conv,A} < G_{conv,B}$

$$T_{s,A} = T_{fb,A} - \frac{Q_{conv,A},(1-\widehat{pLoss}_{conv})}{A_s G_{conv,A}} < T_{fb,B} - \frac{Q_{conv,A},(1-\widehat{pLoss}_{conv})}{A_s G_{conv,A}} < T_{fb,B} - \frac{Q_{conv,B},(1-\widehat{pLoss}_{conv})}{A_s G_{conv,A}} < T_{fb,B} - \frac{Q_{conv,B},(1-\widehat{pLoss}_{conv})}{A_s G_{conv,B}} = T_{s,B} \qquad \text{(S5-50)}$$

Then, $T_{s,A} < T_{s,B}$. In addition, as previously discussed, $T_{fb,A} < T_{fb,B}$ and $T_{c,A} > T_{c,B}$. From the following diagram,



we conclude that $LMTD_A < LMTD_B$.

Now, returning to equation (36) in the main article, we have

$$Q_{conv,A}\big(1 - \widehat{pLoss}_{conv}\big) = U_{conv,A} A_{conv} LMTD_A < U_{conv,B} A_{conv} LMTD_B = Q_{conv,B}\big(1 - \widehat{pLoss}_{conv}\big) \qquad \text{(S5-51)}$$

Thus, $Q_{conv,A} < Q_{conv,B}$, contradicts the result obtained from (S5-20). Therefore, both solutions cannot have different temperature of the oil entering the radiation section (T_c). for the same objective function. In addition, one can also argue by looking into the equations that if T_c is the same for both solutions, then all the other variables are also the same.

Thus, we conclude that the solution is unique.

S6: MINLM with discrete variables

For the MINLM model with discrete variables, the following design variables are needed to be expressed using binary variables : $yL_i, ydo_i, yHs_i, yDs_i$, $yr_{p,i}^{rad}, yr_{p,i}^{conv,h}$, and $yr_{p,i}^{conv,v}$.

- Tube length :

$$L = \sum \hat{L}_i yL_i \quad \text{(S6-1)}$$

$$1 = \sum yL_i \quad \text{(S6-2)}$$

- Tube outside diameter and wall thickness:

$$d_o = \sum \hat{d}_{o,i} ydo_i \ , t_d = \sum \hat{t}_{d,i} ydo_i \quad \text{(S6-3)}$$

$$1 = \sum ydo_i \quad \text{(S6-4)}$$

- Stack height:

$$H_s = \sum \hat{H}_{s,i} yHs_i \quad \text{(S6-5)}$$

$$1 = \sum yHs_i \quad \text{(S6-6)}$$

- Stack diameter and wall thickness:

$$D_s = \sum \hat{D}_{s,i} yDs_i \ , t_s = \sum \hat{t}_{s,i} yDs_i \quad \text{(S6-7)}$$

$$1 = \sum yDs_i \quad \text{(S6-8)}$$

- Pitch ratio:

$$r_p^{rad} = \sum \widehat{r_{p,i}^{rad}}\ yr_{p,i}^{rad} \quad \text{(S6-9)}$$

$$r_p^{conv.h} = \sum \widehat{r_{p,i}^{conv,h}}\ yr_{p,i}^{conv,h} \quad \text{(S6-10)}$$

$$r_p^{conv,v} = \sum \widehat{r_{p,i}^{conv,v}}\ yr_{p,i}^{conv,v} \quad \text{(S6-11)}$$

$$1 = \sum yr_{p,i}^{rad} \quad \text{(S6-12)}$$

$$1 = \sum yr_{p,i}^{conv,h} \tag{S6-13}$$

$$1 = \sum yr_{p,i}^{conv,v} \tag{S6-14}$$

To build this model, the equations of the continuous model (S3) are used. In addition, the following equations are also added:

Geometry equations.

$$d_i = d_o - 2\ t_d \tag{S6-15}$$

$$d_f = d_o + 2\ l_f \tag{S6-16}$$

$$d_c^{rad} = d_o\ r_p^{rad} \tag{S6-17}$$

$$d_c^{conv,h} = d_o\ r_p^{conv,h} \tag{S6-18}$$

$$d_c^{conv,v} = d_o\ r_p^{conv,v} \tag{S6-19}$$

$$Nt_{shield} = Nt_{conv}^{pass} N_{passes} \tag{S6-20}$$

$$Nt_{rad}^{sidewall} = Nt_{rad}^{pass} N_{passes} - Nt_{rad}^{ceil} \tag{S6-21}$$

$$W_{conv} = \left(Nt_{conv}^{pass}\ \ N_{passes} - 1\right) d_c^{conv,h} + \frac{d_c^{conv,h}}{2} + d_f + 2\ \Delta W_{conv} \tag{S6-22}$$

$$W_{rad} = 2\left[\left(\frac{Nt_{rad}^{ceil}}{2}\right) d_c^{rad} + d_o + \Delta W_{rad}\right] + W_{conv} \tag{S6-23}$$

$$H_{rad} = \left(\frac{Nt_{rad}^{sidewall}}{2}\right) d_c^{rad} + d_o + 2\ \Delta H_{rad} \tag{S6-24}$$

$$H_{conv} = (Nr_{conv} + 1)\ d_c^{conv,v} + \frac{d_o}{2} + \frac{d_f}{2} + \Delta H_{conv} \tag{S6-25}$$

$$H = H_{rad} + H_{conv} + H_s \tag{S6-26}$$

$$E_l = L - \hat{k}_l \tag{S6-27}$$

$$1 < H_{rad}/W_{rad} < 1.5 \tag{S6-28}$$

$$1.8 < L/W_{rad} \ < 3 \tag{S6-29}$$

$$Nt_{rad} = Nt_{rad}^{pass} N_{passes} + Nt_{shield} \tag{S6-30}$$

$$Nt_{conv} = Nt_{conv}^{pass} N_{passes} Nr_{conv} \tag{S6-31}$$

$$3.5 < \frac{(L \times W_{rad} \times H_{rad})}{(\pi d_o E_l Nt^{rad})} < 4.5 \tag{S6-32}$$

$$d_c^{rad} \geq d_o^{rad} + \hat{\varepsilon}_r \tag{S6-33}$$

$$d_c^{conv,h} \geq 2\ l_f + d_o^{conv} \tag{S6-34}$$

$$\sqrt{(d_c^{conv,v})^2 + \left(\frac{d_c^{conv,h}}{2}\right)^2} \geq 2\ l_f + d_o^{conv} \tag{S6-35}$$

$$D_s < W_{conv} \tag{S6-36}$$

$$L_{oil} = E_l\left(Nt^{shield}(Nr_{conv} + 1) + Nt_{rad}^{ceil} + Nt_{rad}^{sidewalls}\right) \tag{S6-37}$$

$$A_{rad} = Nt^{rad} E_l\ \pi d_o \tag{S6-38}$$

$$A_s = E_l\ \left\{W_{conv} - Nt_{conv}^{pass} N_{passes}\left[d_o^{conv} + N_f\ (d_f - d_o^{conv}) t_f\right]\right\} \tag{S6-39}$$

Equations of oil velocity, pressure drop and mass flux of gas.

$$F_{oil}^{tube} = \widehat{M}_{Oil}/N_{passes} \tag{S6-40}$$

$$V_{oil}^{tube,rad} = \frac{\widehat{M}_{oil}}{N_{passes}\widehat{\rho}_{oil}\left[\pi\left(d_i^{rad}\right)^2/4\right]} \leq V_{oil,MAX}^{tube} \tag{S6-41}$$

$$V_{oil}^{tube,conv} = \frac{\widehat{M}_{oil}}{N_{passes}\widehat{\rho}_{oil}\left[\pi\left(d_i^{conv}\right)^2/4\right]} \leq V_{oil,MAX}^{tube} \tag{S6-42}$$

$$Re_t^{rad} = \frac{4\widehat{M}_{Oil}}{d_i^{rad} N_{passes}\pi\,\widehat{\mu}_{oil}} \tag{S6-43}$$

$$Re_t^{conv} = \frac{4\widehat{M}_{Oil}}{d_i^{conv} N_{passes}\pi\,\widehat{\mu}_{oil}} \tag{S6-44}$$

$$f_{rad} = 5.5 \times 10^{-3}\left[1 + \left(2 \times 10^4\left(\frac{\hat{r}}{d_i^{rad}}\right) + \frac{10^6}{Re_t^{rad}}\right)^{1/3}\right] \tag{S6-45}$$

$$f_{conv} = 5.5 \times 10^{-3}\left[1 + \left(2 \times 10^4\left(\frac{\hat{r}}{d_i^{conv}}\right) + \frac{10^6}{Re_t^{conv}}\right)^{1/3}\right] \tag{S6-46}$$

$$\Delta P_{oil}^{tube,rad} = f\,\frac{L_{oil}}{\left(d_i^{rad}\right)}\left(\frac{\widehat{\rho}_{oil}\left(V_{oil}^{tube,rad}\right)^2}{2}\right) \tag{S6-47}$$

$$\Delta P_{oil}^{tube,conv} = f\,\frac{L_{oil}}{\left(d_i^{conv}\right)}\left(\frac{\widehat{\rho}_{oil}\left(V_{oil}^{tube,conv}\right)^2}{2}\right) \tag{S6-48}$$

$$\Delta P_{oil}^{tube,rad} + \Delta P_{oil}^{tube,conv} \leq \Delta P_{oil,Max}^{tube} \tag{S6-49}$$

$$M_{gas} = \frac{Q_{Rad}(1 - \widehat{percLoss}_{Rad})}{\widehat{cp}g_1\left(\hat{T}_{flame} - \hat{T}_{fb}\right)} \tag{S6-50}$$

$$G_{conv} = \frac{M_{gas}}{A_s} \tag{S6-51}$$

$$0.3 < G_{conv} < 0.4 \tag{S6-52}$$

Stack equations

$$\rho_{fb} = \frac{P}{g \cdot T_{fb}} \cdot MW_{avg} \tag{S6-53}$$

$$\rho_m = \frac{P}{g \cdot \left(\frac{T_S + T_{fb}}{2}\right)} \cdot MW_{avg} \tag{S6-54}$$

$$\rho_{stack} = \frac{P}{g \cdot T_{stack}} \cdot MW_{avg} \tag{S6-55}$$

$$C_2 = 0.075 + 1.85 Re^{-0.3} = 0.075 + 1.85 \left(\frac{d_o\, G_{conv}}{\hat{\mu}_{gas}}\right)^{-0.3} \tag{S6-56}$$

$$C_4 = 0.11 \left(0.05 \frac{d_c^{conv,h}}{d_0}\right) m \tag{S6-57}$$

$$m = -0.7 \left(\frac{l_f}{s}\right)^{0.2} \tag{S6-58}$$

$$C_6 = 1.11 + \left[1.8 - 2.1 e^{-0.15 N_f{}^2}\right] \mathrm{e}^{-2\left(\frac{d_c^{conv,v}}{d_c^{conv,h}}\right)} - \left[0.7 - 0.8 \mathrm{e}^{-0.15 N_f{}^2}\right] \mathrm{e}^{-0.6\left(\frac{d_c^{conv,v}}{d_c^{conv,h}}\right)} \tag{S6-59}$$

$$f = C_2 C_4 C_6 \left(\frac{d_f}{d_o}\right) \tag{S6-60}$$

$$\beta = \frac{A_S}{W_{conv}\, E_l} \tag{S6-61}$$

$$a = \frac{1+\beta^2}{4\, Nr_{conv}} \rho_m \left(\frac{1}{\rho_{fb}} - \frac{1}{\rho_s}\right) \tag{S6-62}$$

$$\Delta p_f = (f + a) Nr_{conv} \frac{G_{conv}^2}{\rho_m} \tag{S6-63}$$

$$A_{s,shield} = E_l \left\{W_{conv} - Nt_{conv}^{pass} N_{passes}\, d_o^{conv}\right\} \tag{S6-64}$$

$$G_{shield} = \frac{M_{gas}}{A_{s,shield}} \tag{S6-65}$$

$$\Delta p_{shield} = 0.2 \frac{\left(G_{shield}\right)^2}{2\rho_{fb}} \tag{S6-66}$$

$$\Delta p_{conv} = \Delta p_{shield} + \Delta p_f \quad \text{(S6-67)}$$

$$G_{stack} = \left(\frac{M_{gas}}{\frac{\pi D_S^2}{4}}\right) \quad \text{(S6-68)}$$

$$\Delta p_{stack} = 2.76 \cdot 10^{-5} \left(\frac{\left(G_{stack}\right)^2 T_{stack}}{D_S}\right) H_s \quad \text{(S6-69)}$$

$$\Delta p_{minor} = 3\frac{\left(G_{stack}\right)^2}{2\rho_{stack}} \quad \text{(S6-70)}$$

$$\sum F \quad = \Delta p_{conv} + \Delta p_{stack} + \Delta p_{minor} \quad \text{(S6-71)}$$

$$\hat{\rho}_{g,o} = \frac{P}{g \cdot T_{flame}} \cdot MW_{avg} \quad \text{(S6-72)}$$

$$g\left(\rho_a - \rho_{gas,avg}\right)H \geq (1+\varepsilon)\left(\widehat{\Delta P}_b + \sum F \quad\right) \quad \text{(S6-73)}$$

S7. Nomenclature

In this work, all parameters show with "^" on top. The rest are variables.

α	: gas absorptivity
$\hat{\delta}_{H,1}, \hat{\delta}_{H,2}$	: coefficient for the clearances at the end of the height
$\hat{\delta}_{W,1}, \hat{\delta}_{W,2}$	: coefficient for the clearances at the end of the ceiling
$\hat{c}_{\alpha,i}$	: coefficients for calculating the gas absorptivity (*i*=1~3)
$\hat{c}_{F,i}$	: coefficients for calculating the exchange factor (*i*=1~6)
$\hat{c}_{e,i}$	: coefficients for calculating the emissivity (*i*=1~4)
$\hat{c}_{PL,i}$	: coefficients for calculating the PL factor with the fixed excess air (*i*=1~4)
$\widehat{co}_{h,i}$	: coefficients for the enthalpy as a function of temperature (*i*=1~3)
A_{cp}	: area of cold planes
A_{conv}	: convection heat transfer area
A_{cp}^{Shield}	: area of cold planes for Shield
A_{cp}^{wall}	: area of cold planes for wall
A_{Rad}^{T}	: tube radiation area
A_s	: flow area in the central place of a tube row

Ab	: exposed area of the root tube
Aof	: area of the fins
Aot	: heat transfer area per unit length of the finned tubes
c_{cost}	: unitary cost of convection section
cpg_l	: heat capacity
C_{cost}	: cost of the convection coil
CRF	: capital recovery cost
d_i	: tube inside diameter
d_f	: fin diameter
$d_c^{conv,h}$	: horizontal distance between finned tube centers in the convection section
$d_c^{conv,v}$	: vertical distance between finned tube centers in the convection section
d_c^{rad}	: distance between tube centers in the radiant section
d_o	: tube outside diameter
d_r	: distance between rows in the convection section
D_s	: stack diameter
e_r	: emissivity
E_l	: tube exposed length
$\widehat{ex_{air}}$	: percent excess air
f	: Dracy friction factor
f_u	: correction factor
F	: friction loss
F_{oil}^{tube}	: flowrate of oil in each tube
$Flux$	: flux
FB_{cost}	: cost of the firebox
G	: gas mass velocity at cross-section length
G_{stack}	: mass flux of the flue gas in the stack
ΔH	: clearance at the end of the height
H	: total height
H_{conv}	: height of convection section
H_{rad}	: height of radiant section
H_s	: height of stack section
ΔH_{conv}	: vertical clearance at the end of the ceiling in the convection section
ΔH_{rad}	: vertical clearance at the end of the ceiling in the radiation section
hct	: inner side heat transfer coefficient
h_{fb}^{gas}	: enthalpy of gas
$\hat{h}_{flame}^{gas}$	: enthalpy of flame
h_s^{gas}	: enthalpy of the gas leaving the convection section
h_s	: enthalpy of stack
h_{Ci}^{HT}	: convective heat transfer coefficient inside the tube
h_{Co}^{HT}	: convective heat transfer coefficient outside the tube
$\hat{h}_O^{oil}$	: enthalpy of the oil at the desired outlet temperature
j	: factor for triangular layout of tubes in the convection section
$\hat{k}_l$	: unexposed length of the tube
$\hat{k}_{fin}$	: Thermal conductivity of the fin
$\hat{k}_s$	: Thermal conductivity of the pipe wall
$\hat{k}_t$	: Thermal conductivity of the tube material
$\hat{k}_{oil}$	: Thermal conductivity of oil
L	: length

l_f	: fin height
L_{oil}	: length of oil tube
$LMTD$	: logarithm means temperature difference from flue gas to fluid
$\widehat{M}_{oil}$	: oil mass flowrate
M_{gas}	: gas mass flowrate
M_{fuel}	: flowrate of fuel
$Ntpass$	: number of total passes
N_f	: number of fins per meter
Nt_{rad}^{ceil}	: number of ceiling tubes
Nt_{shield}	: number of shield tubes
Nt_{rad}	: number of tubes in the radiation section
Nt_{conv}	: number of tubes in the convection section
N_{passes}	: number of passes
Nt_{rad}^{pass}	: number of tubes per pass in the ceiling and wall side
Nt_{conv}^{pass}	: number of tubes per pass in the convection section
Nr_{conv}	: number of rows of tubes in the convection section
ηf	: fin efficiency
ηt	: overall efficiency of the finned surface
o_{cost}	: unitary cost of fuel
O_{cost}	: Operating cost
OT	: plant operating time
$\widehat{pLoss}_{rad}$	: percentage lost through the walls of the furnace
$\widehat{pLoss}_{conv}$	: percentage lost through the convection section
ΔP_{oil}^{tube}	: pressure drop in the tube
$\widehat{\Delta P}_b$	: pressure drop across the burner
Δp_{conv}	: pressure drop in the convection section
Δp_f	: pressure drop across the finned horizontal tubes
Δp_{minor}	: minor loss of pressure
Δp_{rad}	: pressure drop in the radiant section
Δp_{shield}	: pressure drop across the shield tubes
Δp_{stack}	: pressure drop in the stack section
Q_{rad}	: radiation heat
Q_{conv}	: convection heat
Q_{oil}	: heat of oil
Q_s	: heat of gas leaving the convection section
Q_n	: net heat
r_{cost}	: unitary cost of radiant section
R_{cost}	: cost of the radiant coil
$\widehat{Rft}_{max}$	: maximum value of the fouling factor for process stream
$\widehat{Rfgas}$	: fouling factor of the flue gas
Ret	: Reynolds number
$r_p^{conv,h}$	: horizontal pitch ratio of the convection section
$r_p^{conv,v}$	: vertical pitch ration of the convection section
r_p^{rad}	: pitch ratio of the radiation section
s	: clearance between fins
$\hat{T}_a$	: ambient temperature
$\hat{T}_i$	: oil inlet temperature

t_f : fin thickness
$\hat{T}_o$: oil outlet temperature
$\hat{T}_{flame}$: flame temperature
T_c : cross-over oil temperature
T_{fb} : firebox temperature
T_g : temperature of the gas entering the convection section
T_s : temperature of gas leaving the convection section
T_{stack} : mean temperature of the flue gas along the stack
T_w : outside tube wall temperature
Δt : temperature difference between the firebox temperature and oil inlet temperature
T_{cost} : Total cost
t_d : tube wall thickness
t_s : stack wall thickness
U_{conv} : overall heat transfer coefficient for the convection section
$U_{rad,w}$: overall heat transfer coefficient for the radiation section
V_{oil}^{tube} : oil velocity in the tube
$\dot{v}_g$: volumetric flow of gas
V_s : stack velocity
W_{conv} : width of convection section
W_{rad} : width of radiant section
ΔW_{conv} : horizontal clearance at the end of the ceiling in the convection section
ΔW_{rad} : horizontal clearance at the end of the ceiling in the radiation section
$\hat{\rho}_{oil}$: oil density
ρ_{fb} : density of the gas at the temperature of the firebox
$\hat{\rho}_{g,o}$: density of the gas outside of the burner
ρ_m : mean density of the flue gas in the convection section
ρ_s : density of the gas at the convection section
ρ_{stack} : mean density of the gas along the stack
$\hat{\mu}_{gas}$: gas viscosity
$\hat{\mu}_{oil}$: oil viscosity